Macroeconomics – Political Science Forlì

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Website of the course: http://macroeconomics-forli.weebly.com/

Password: macroSID2019

Exercise Lesson:
5 March
12 March
9 April
30 April
14 May
28 May

Mid-term exams:
19 March
11 April
16 May

Macroeconomics Exercise Lesson 3 (Ch. 8 and 9)

Economic Growth I (Ch. 8)

Economic Growth II (Ch. 9)

Ch. 8: Economic Growth I

Key- concepts

- The Solow model (basic version, without technological progress and population growth): accumulation of capital and steady state
- The Golden rule (maximization of the consumption level in the steady state)
- The Solow model in presence of population growth.
- The Solow model in presence of technological progress.

The production function

- Neoclassical production function, with two inputs (K and L)

E.g.: Cobb-Douglas → $Y = K^{\alpha}L^{\beta}$

- Decreasing marginal productivity of capital and labor

- Constant returns to scale (CRTS)

Inputs and output change exactly in the same proportion:

$$zY = F(zK, zL)$$

Under Cobb-Douglas, CRTS implies $\beta+\alpha=1 \Rightarrow Y=K^{\alpha}L^{1-\alpha}$

Per-capita variables

All the variables can be expressed in per-capita terms (in that case, they are denoted by the corresponding small letter):

$$k = K/L$$
$$y = Y/L$$
$$c = C/L$$
$$i = I/L$$

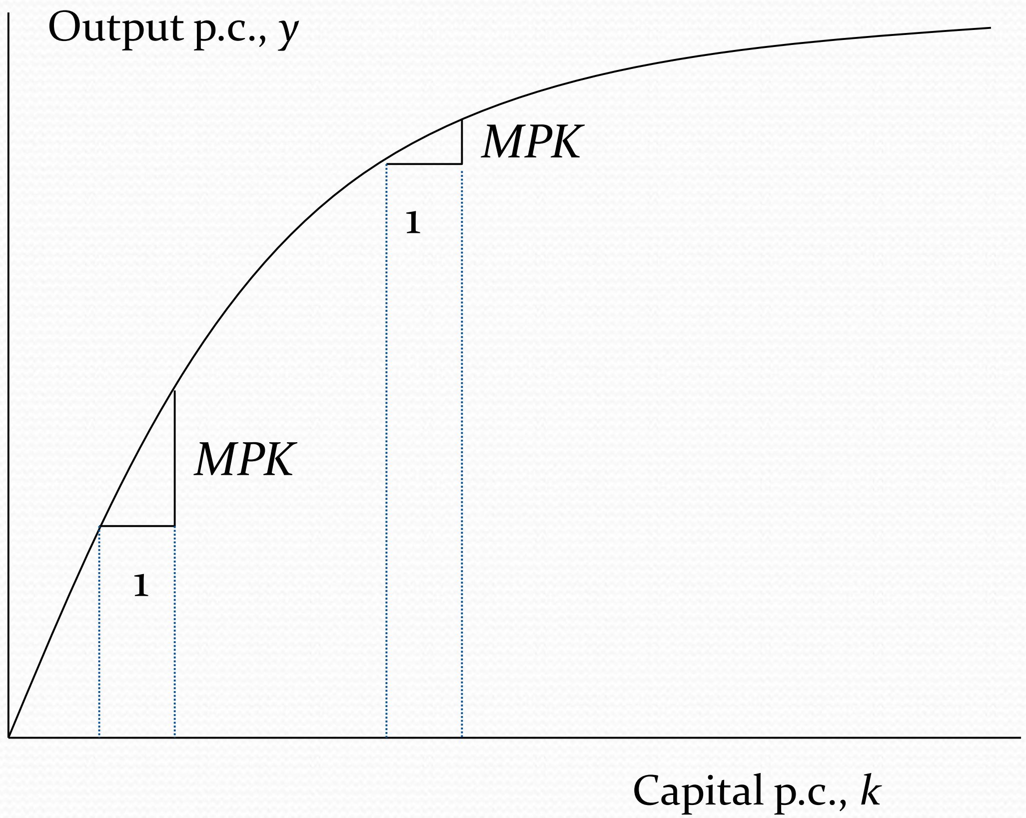
Income and capital per capita (p.c.) represents the average level of income and capital per each individual among the population

By using per-capita variables, it is possible to run cross-country comparisons, even if countries have different size

The per-capita production function

The slope of the curve represents the marginal productivity of capital (MPK), which is decreasing in the amount of capital.

Hence, the slope of the production function declines with the amount of capital used in production



Consumption and Investment functions

Saving (p.c.) is a constant fraction of the disposable income (p.c.), and is denoted as **s (= saving rate)**

Consumption (p.c.) is the remaining fraction of the disposable income (p.c.), and is given by: $c = (1 - s)y$

As in any static model, also in the dynamic model of Solow macroeconomic equilibrium requires that **Investment** (p.c.) equals **Saving** (p.c), i.e.: $i = s\,y = s\,f(k)$

The graph is the same as the production function, simply "re-scaled" by a coefficient that is between zero and one (this coefficient being exactly the saving rate s)

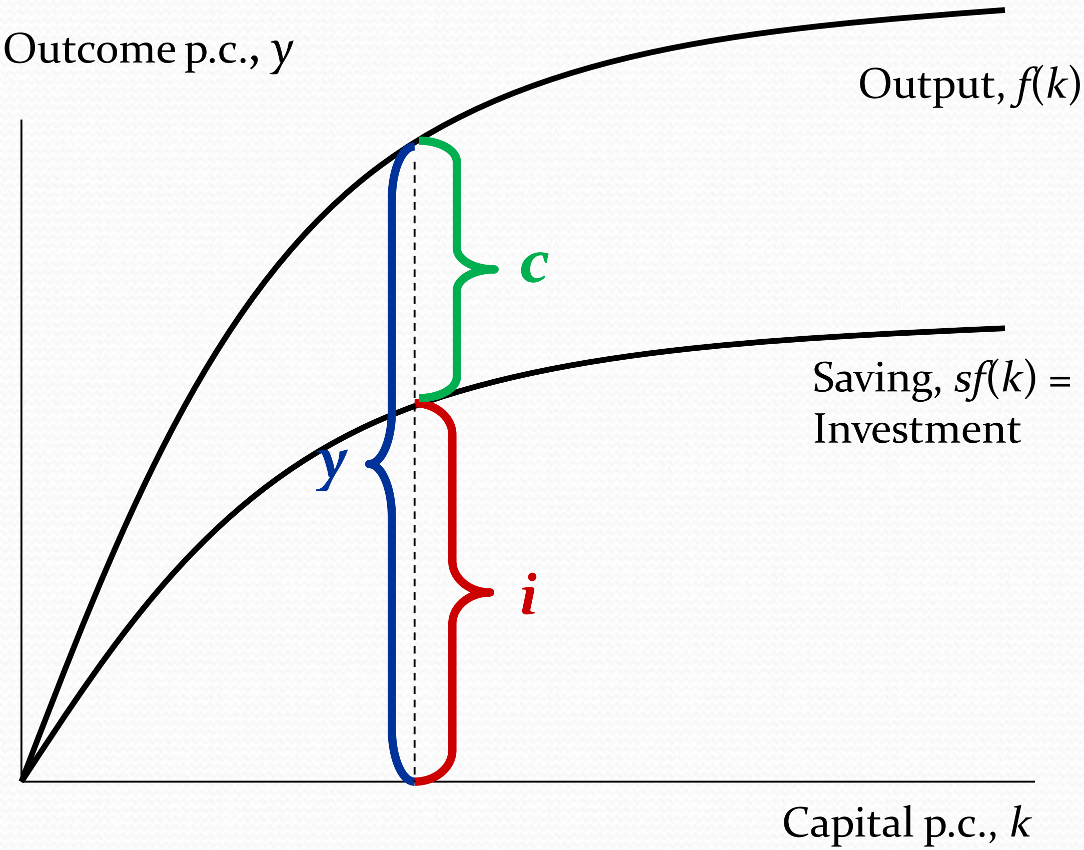
Production, saving and investment

Income (*y*) is split in two parts: consumption (*c*) and saving (*i*)

A change in the saving rate *s* entails shifting the function *sf(k)*:

- upwards (if the change is positive)
- downwards (if negative)

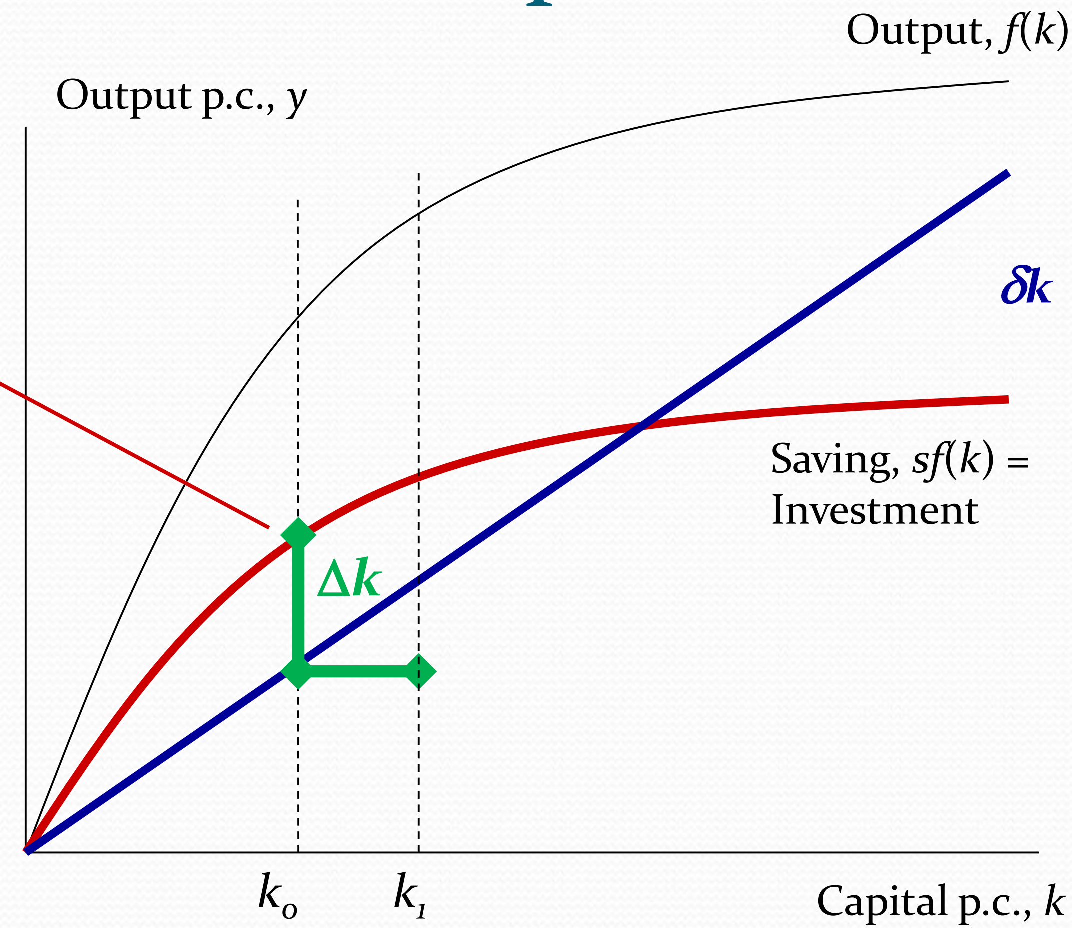
If $s = 1$, all income is saved and $c = 0$



The accumulation of capital

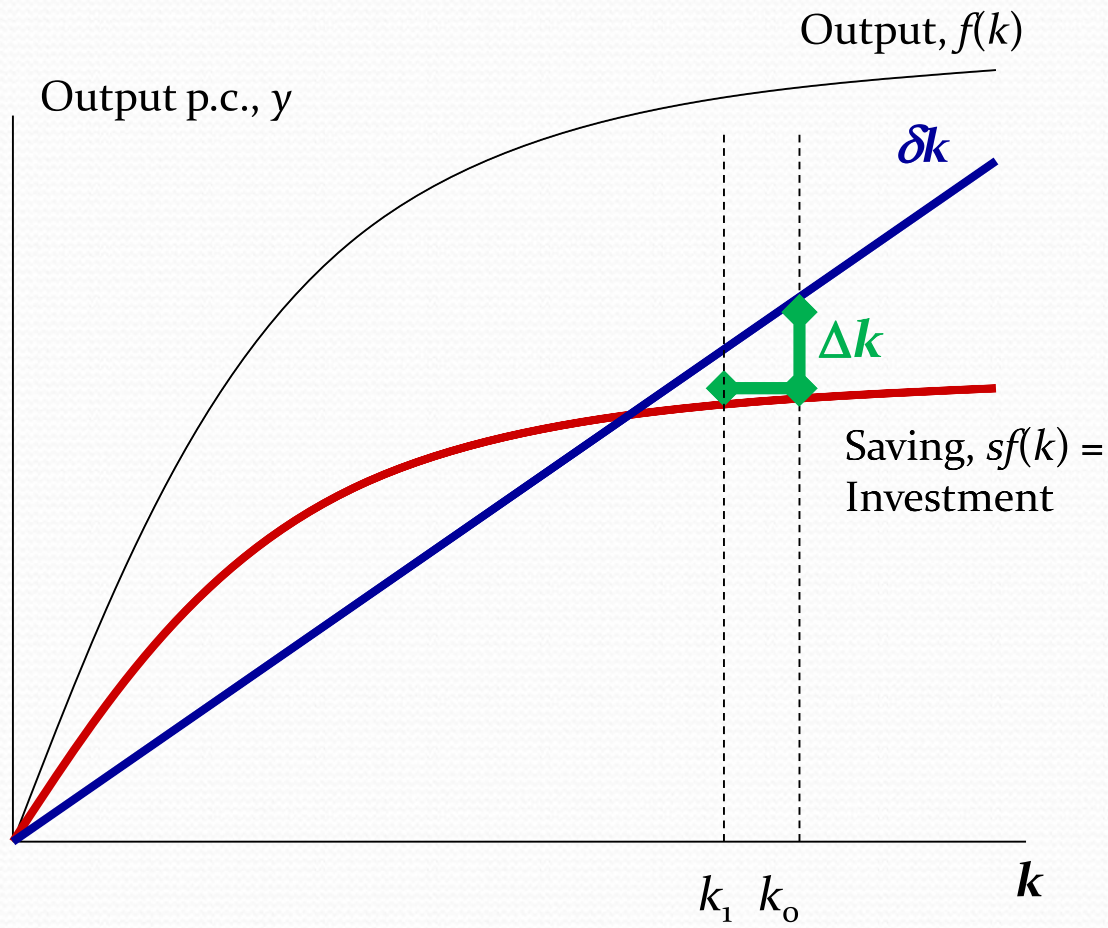
Investment INCREASES the stock of capital per worker for the next period

The DIFFERENCE between investment and depreciation is a measure of the change in the capital stock: it may be positive…



The accumulation of capital

… or it may be negative (in case depreciation exceeds investment)



The steady state

When investment is exactly equal to the depreciation of the capital stock, there is no change in the level of capital p.c.: newly created capital (by investment) is just sufficient to compensate for the loss of capital due to depreciation

In the long run, the economy tends to converge towards a **steady state equilibrium**, in which the endogenous variable k^* does not vary over time

This implies that also income and consumption p.c. do not vary:

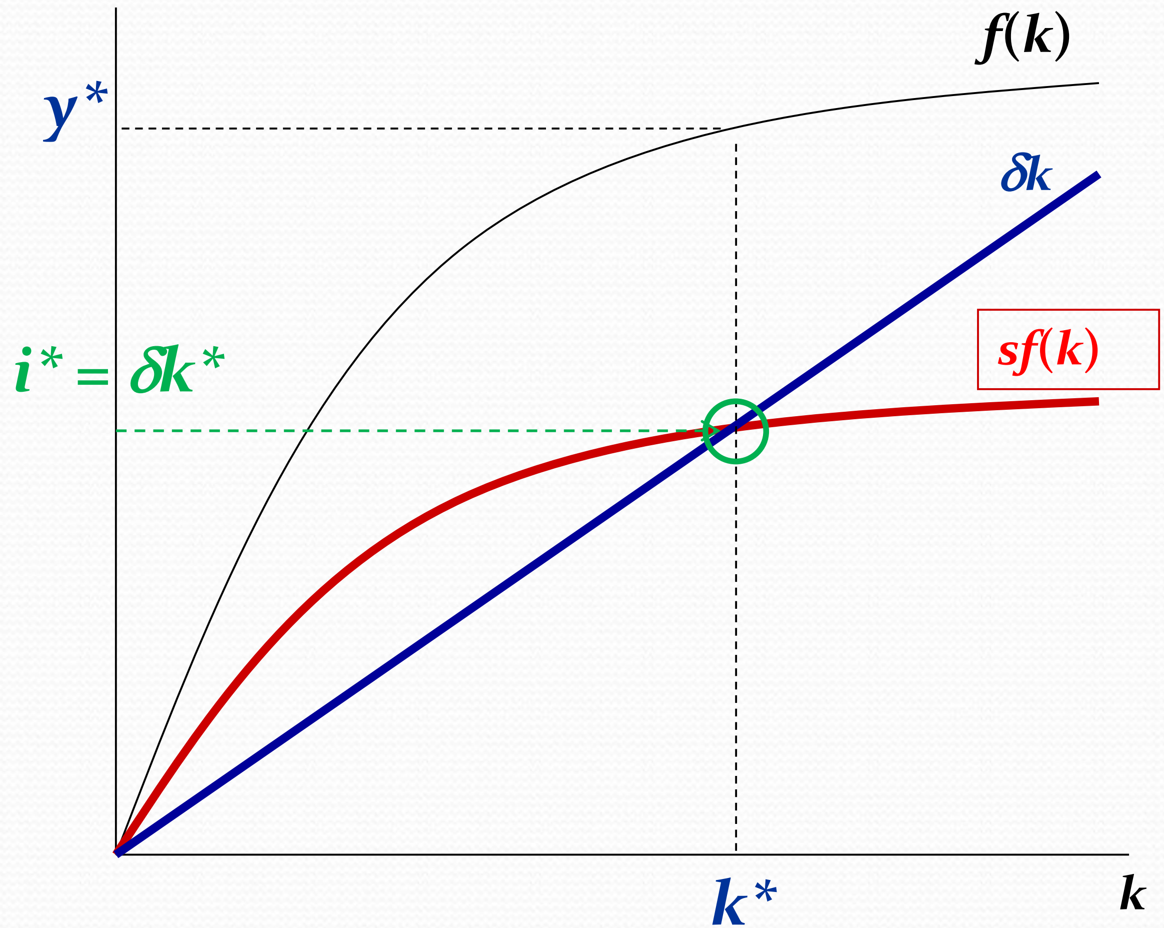
$$y^* = f(k^*)$$

$$c^* = (1-s)\, f(k^*)$$

The steady state in the graph

In the steady state equilibrium, investment (and therefore saving) corresponds to the amount of the former capital stock lost because of depreciation

The level of capital per capita stops growing



Steady State Equilibrium

The steady state equilibium is characterized by: $\Delta k = 0$

Since the law of motion of capital is given by:

$$\Delta k = s f(k) - \delta k,$$

it follows that:

$$0 = s f(k^*) - \delta k^*,$$

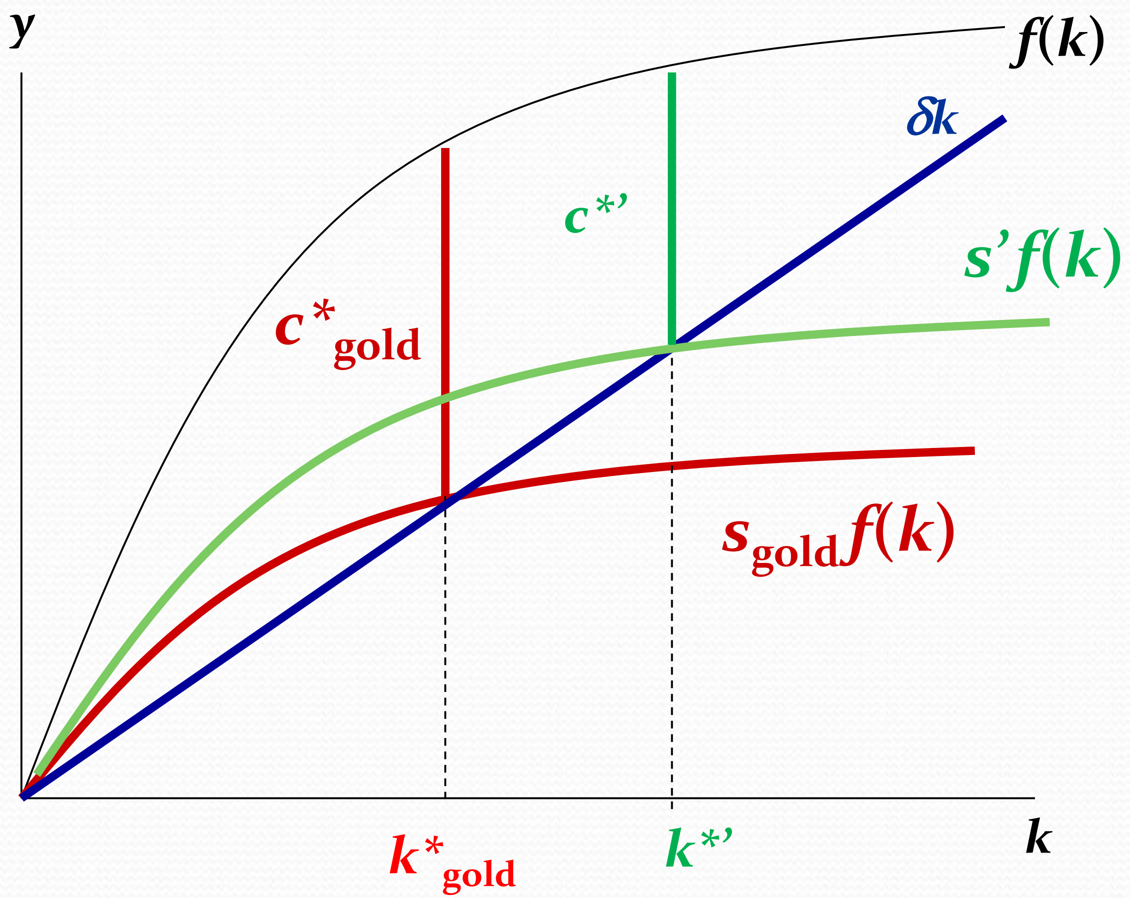
which can be re-written as:

$$k^*/f(k^*) = s/\delta$$

The golden rule

Given δ, different levels of the saving rate *s* lead to different steady state levels of capital and income p.c.

Which of these steady state equilibria is the most preferable? Well, the one in which consumption p.c. turns out to be the highest, so that social welfare is maximized!

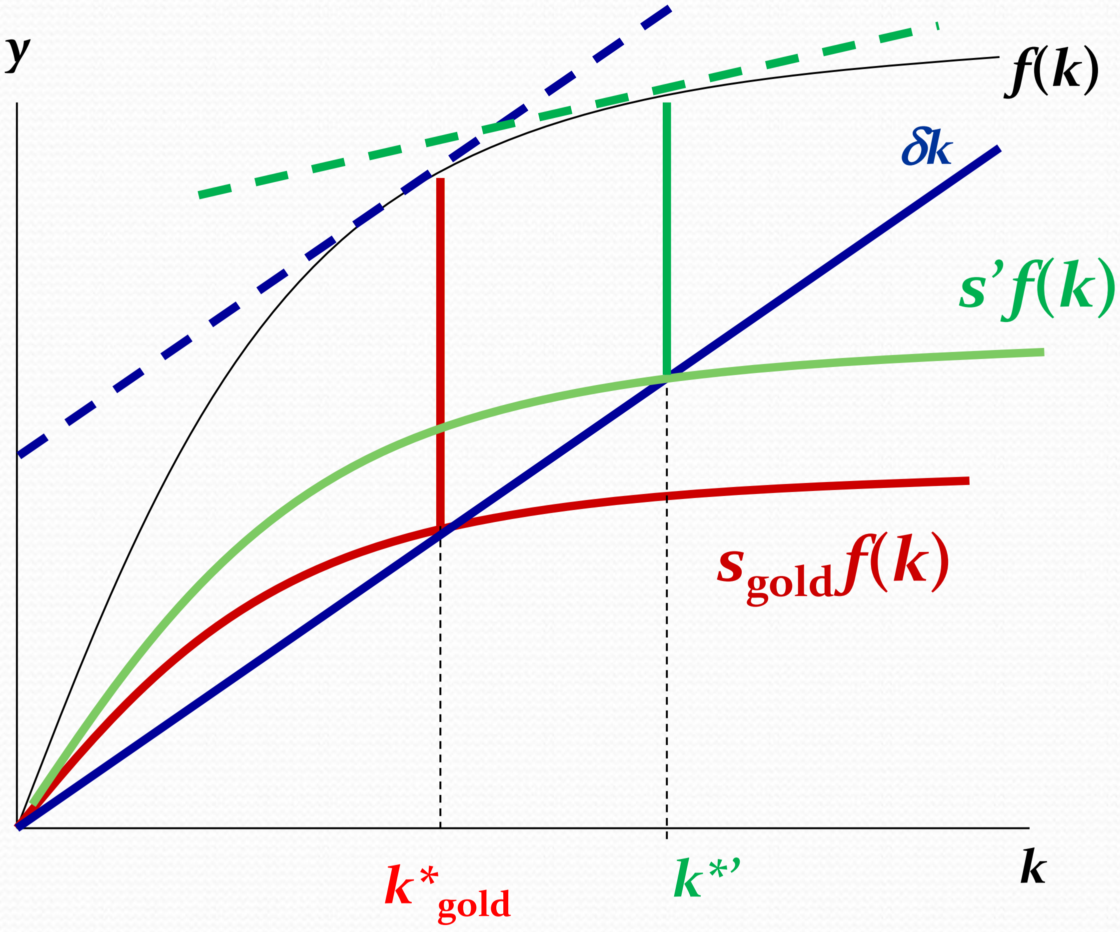


Many equilibium values k^*, but only one is k^*_{gold}

The golden rule

In the graph, the steady state golden rule is achieved when the slope of the production function is equal to the slope of the line representing depreciation:

so, $MPK = \delta$



Convergence and golden rule

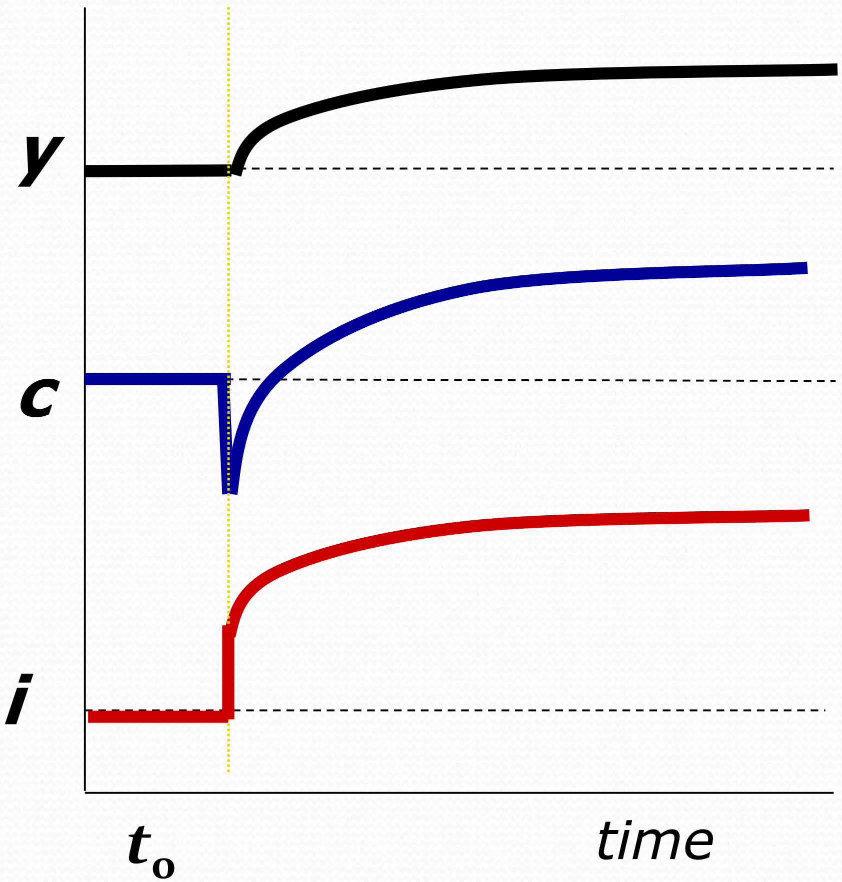
The economic system does not tend to converge spontanously to the golden-rule level of capital p.c.

Only in the case that the saving rate is compatible with k^*_{gold}, consumption p.c. is actually maximized in the long run.

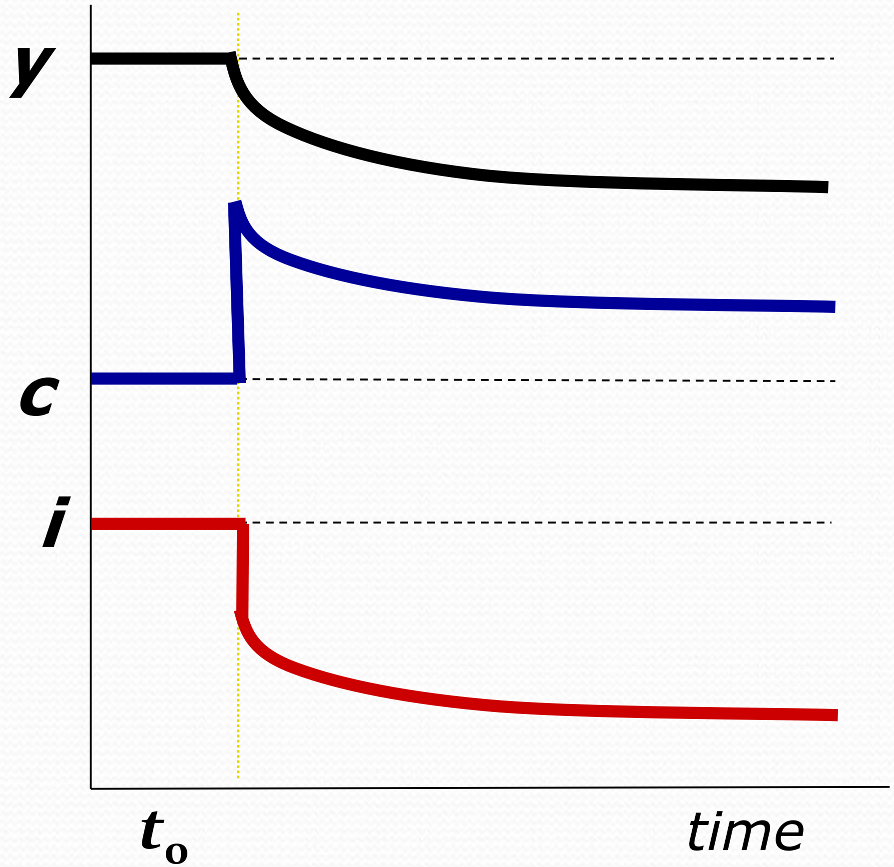
If it is not the case, then achieving the steady state equilibrium of golden rule requires a change in the saving rate.

What happens in the aftermath of a change in the saving rate, and therefore along the transition path towards the new steady state equilibrium?

Initial ***k*** is too low: $k^* < k^*_{gold}$



Initial ***k*** is too high: $k^* > k^*_{gold}$



Exercise 1

Draw the graph of the Solow model.

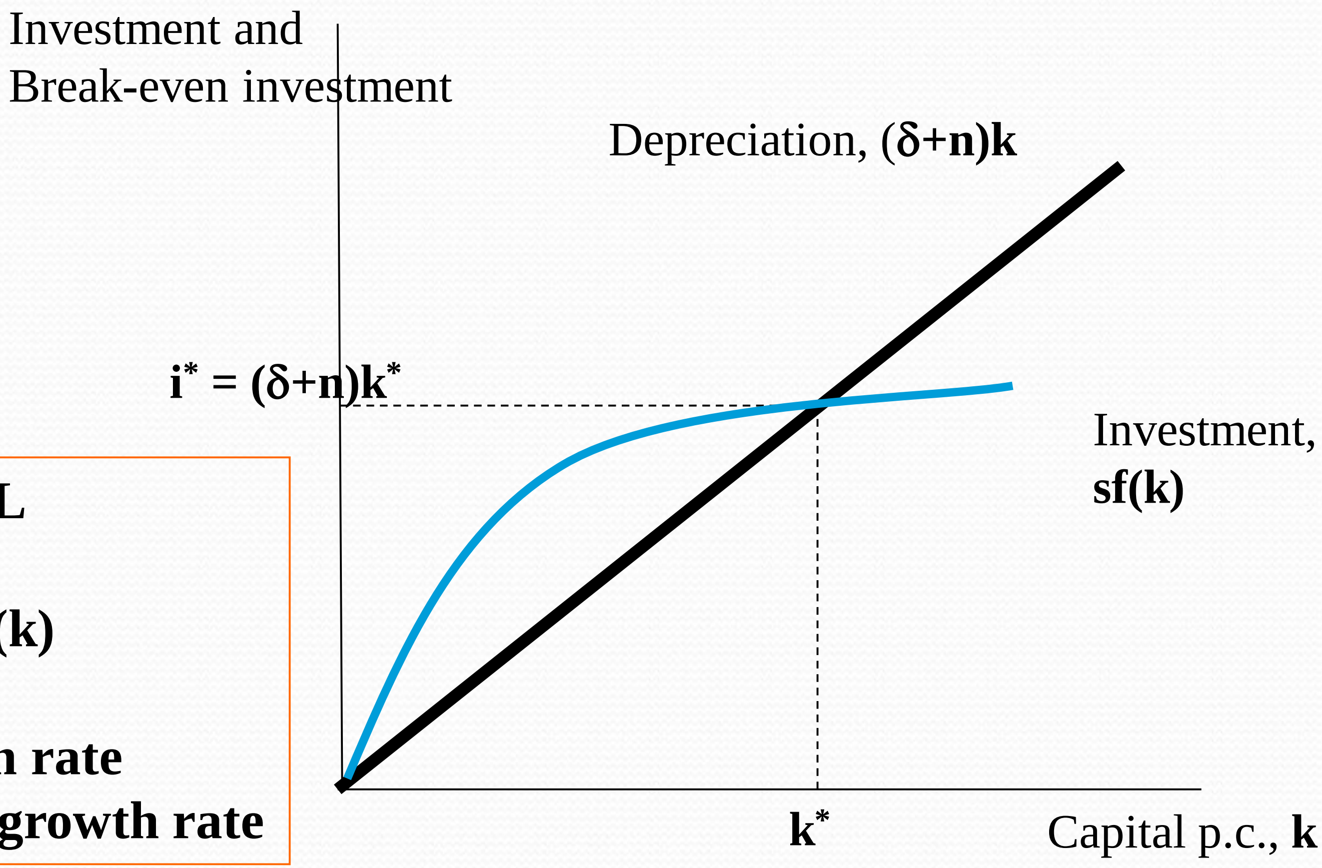
a) Identify the steady state equilibrium in presence of population growth.

b) Show how this equilibrium is modified in case of a reduction in the depreciation rate of capital. Explain what happens from an economic standpoint.

c) Show how the steady state equilibrium is modified in case of an increase in the population growth rate and provide an economic interpretation.

d) Explain whether the theoretical predictions of the Solow model are supported by some empirical evidence.

Es.1 – point (a)



k = K/L, y = Y/L
y = f(k)
i = I/L = sy = sf(k)
s = saving rate
δ = depreciation rate
n = population growth rate

Es.1 – point (a)

The steady state is a long-run condition of the economic system, in which both the capital stock and the level of output per capita are constant over time (i.e. they do not vary)

$$\Delta k = i - \delta k - nk = 0$$

Since $i = s\,f(k)$ $\Rightarrow$ $\Delta k = s\,f(k) - \delta k - nk = 0$

$$\Rightarrow \quad s\,f(k) = (\delta + n)k$$

In order for the capital stock p.c. to be constant over time, new investment must:

- provide capital to replace the amount of the former capital endowment which is lost because of depreciation (i.e. δk)
- provide new workers with the same capital stock p.c. as the old number of workers (nk)

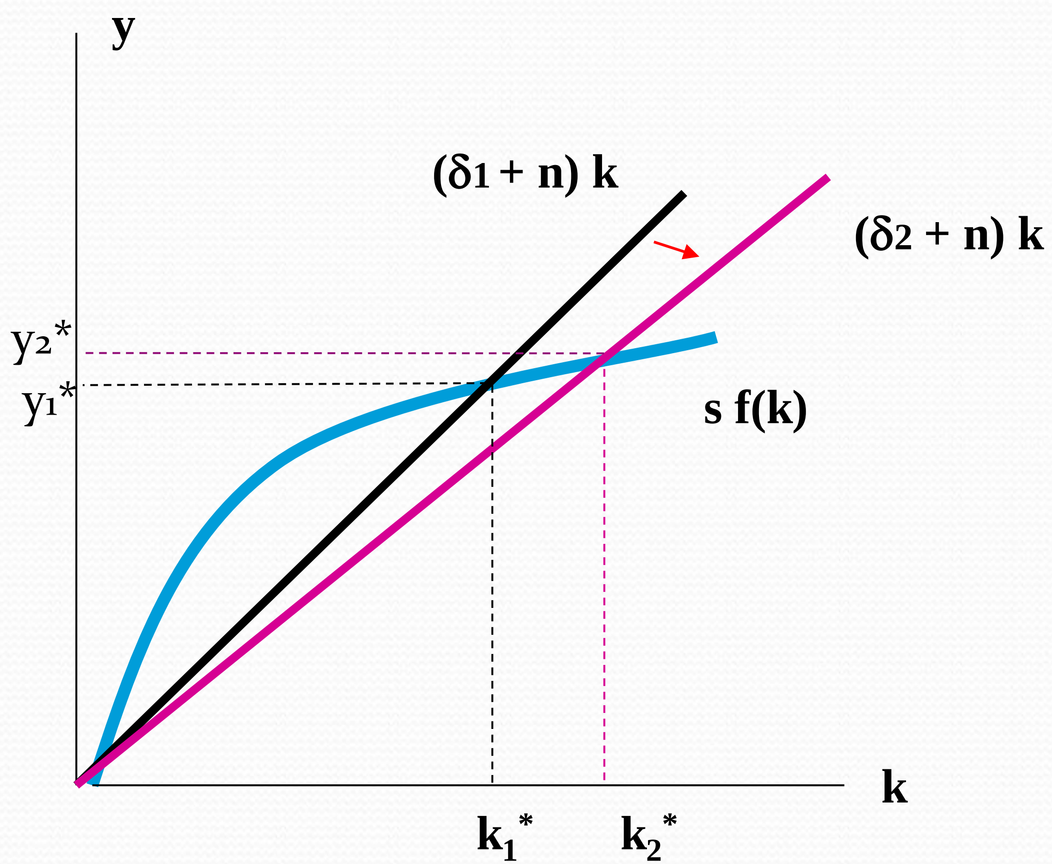
Es. 1 – point (b)

What if δ decreases?

New value: $\delta_2 < \delta_1$

At k_1*, the amount of capital that deteriorates because of depreciation, plus the one needed to equip new workers, is lower than the amount of capital newly created by investment
$\Rightarrow$ k increases up to the new steady state level k_2*, where $k_2^* > k_1^*$.

The new steady state level of income p.c., namely y_2*, will be higher, too: $y_2^* > y_1^*$

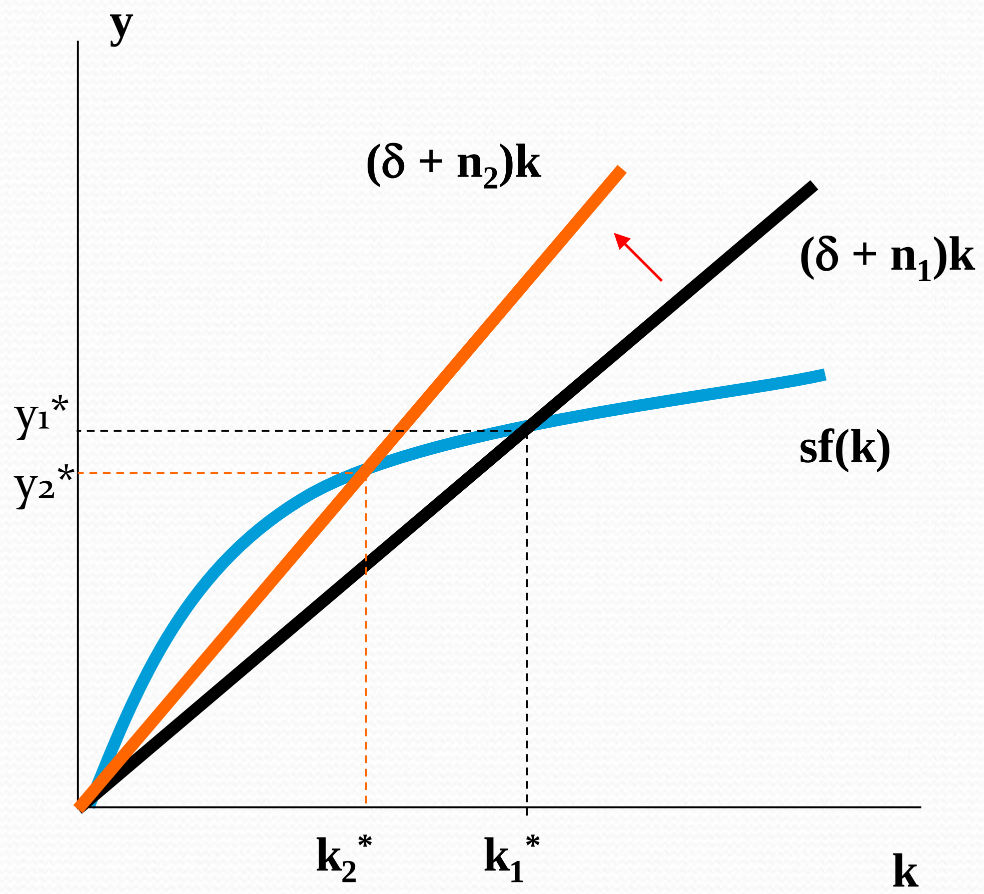


Es.1 – point (c)

What if n increases?
New value: $n_2 > n_1$

At k_1^*, new investment will be lower than the sum of the capital deteriorated and the capital needed to equip new workers ⇒ k decreases until the new steady-state level, denoted by k_2^* (where $k_2^* < k_1^*$)

At k_2^*, the equilibrium income p.c., namely y_2^*, will be lower: $y_2^* < y_1^*$



Es. 1 – point (d)

Model's prediction: if the growth rate of population increases, the steady state level of the capital p.c. will decrease

If one observes the relationship between data on income p.c and population growth rates for many countries all around the world, it can be noticed that countries in which the population tends to grow faster are actually those with lower levels of income p.c.

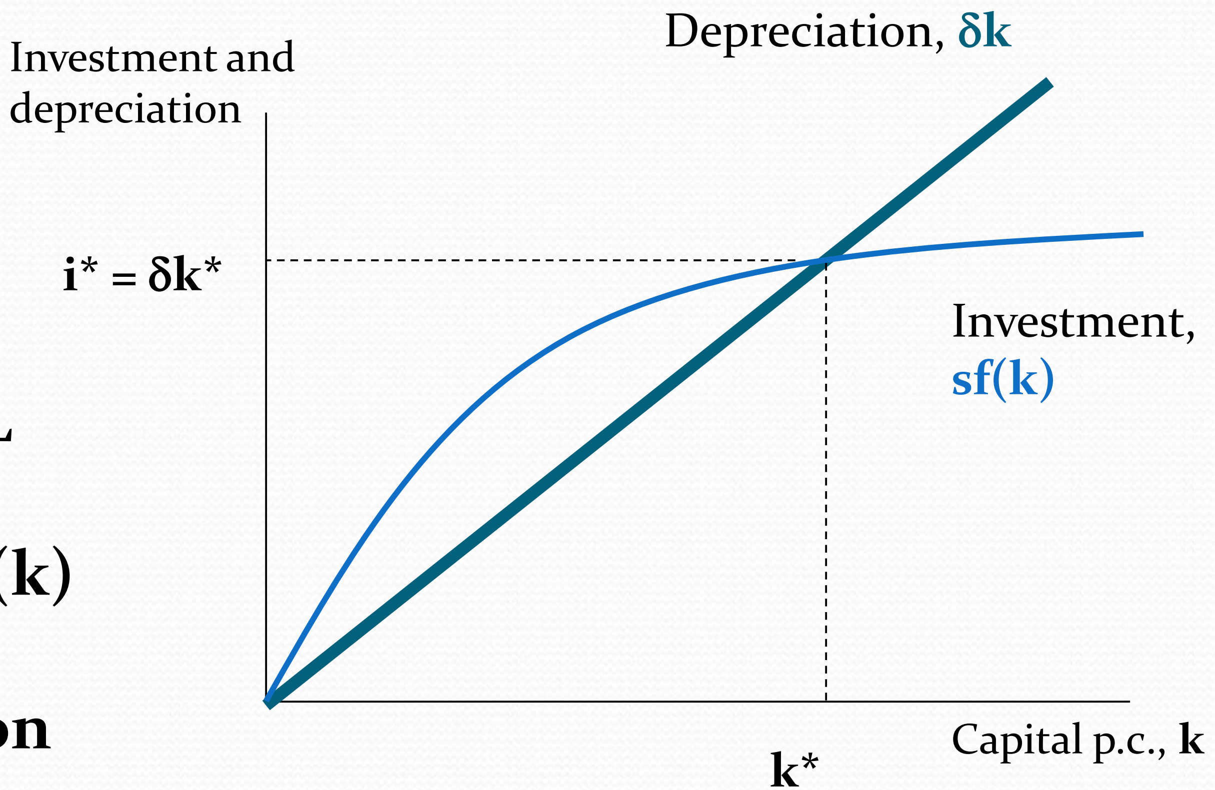
- This prediction of the Solow model seems to be supported by the empirical evidence.
- However, be careful when interpreting economic data: there might be multiple explanations for the same phenomenon!

Exercise 2

Draw the graph of the Solow model assuming - for the sake of simplicity- no population growth (i.e. n=0).

- Identify the steady state equilibrium (simplified version of exercise 1)
- Define and identify the steady state level of capital p.c. under the golden rule

Es. 2 – point (a)



- **$k = K/L$, $y = Y/L$**
- **$y = f(k)$**
- **$i = I/L = sy = sf(k)$**
- **s = saving rate**
- **δ = depreciation rate**

Es. 2 – point (b)

In a closed economy with no population growth and no technical progress, the **steady state** is a long-run condition in which capital stock and income p.c. are constant over time:

$$\Delta k = i - \delta k = s\,f(k) - \delta k = 0 \;\Leftrightarrow\; k^*/f(k^*) = s/\delta$$

Es. Cobb-Douglas production function: $Y = K^{\alpha} L^{(1-\alpha)}$

$\Rightarrow Y/L = K^{\alpha} L^{(1-\alpha)} / L = (K^{\alpha}/L^{\alpha})(L^{(1-\alpha)}/ L^{(1-\alpha)})$

$\Rightarrow y = f(k) = k^{\alpha}$

$\Rightarrow$ steady state: $k^*/(k^*)^{\alpha} = s/\delta$

Es. 2 – point (b)

max c = max [f(k) - δk]

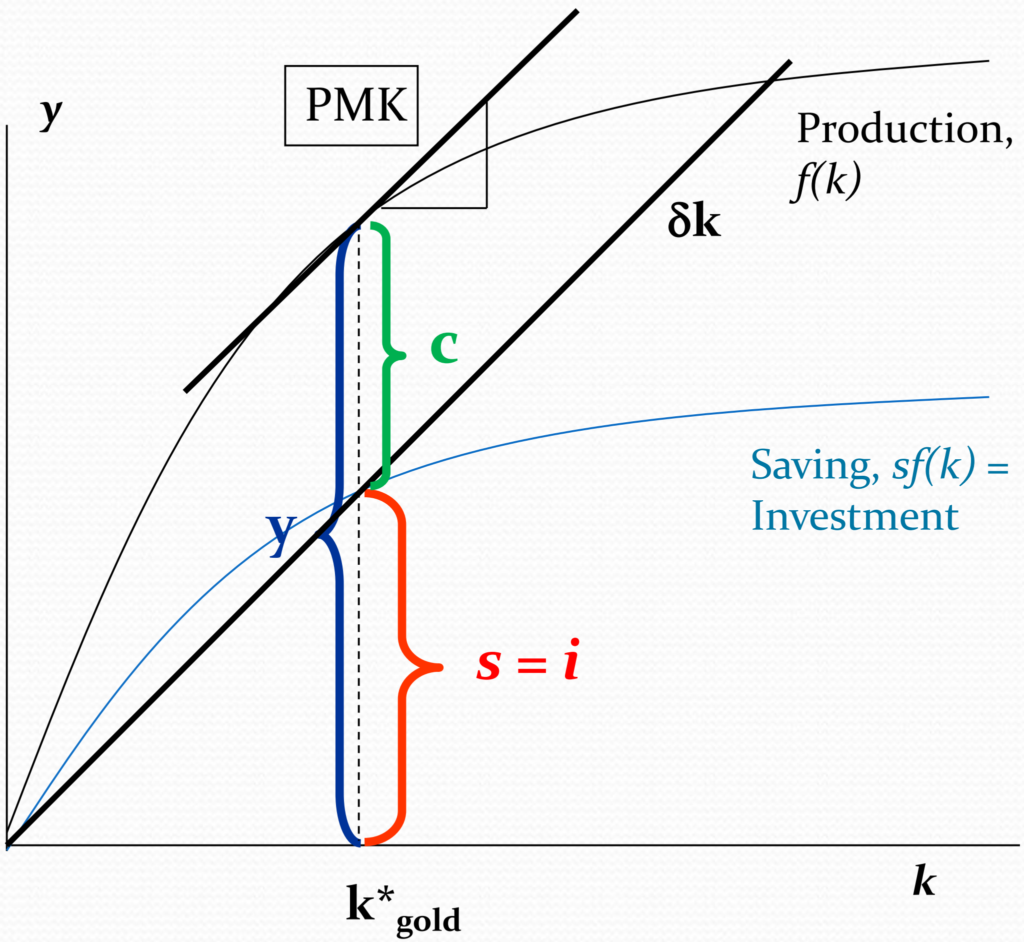
where $\delta k = sf(k)$ since we assume to be in the steady state equilibrium

Condition: **PMK - δ = 0**

where PMK= $f'(k) = df(k)/dk$

⇒ **PMK = δ**

k^*_{gold} is that level of k^* where PMK = δ



Again on the golden rule

The golden rule allows for the identification of the particular steady state equilibrium in which the level of consumption is maximized, and therefore, social welfare is at the maximum.

k* of *golden rule* (i.e. k^*_{gold}) is that particular level of k which leads to:

***max* c = *max* [income p.c. – saving p.c.] =**

$$= \max [f(k) - s f(k)]$$

given the constraint represented by the steady state condition (i.e. s f(k) = δk)

$$\Rightarrow k^*_{gold}: \max c = \max [f(k) - \delta k]$$

Maximization

A generic function g(x) is maximized by looking for the particular value of the variable x at which the derivative of the function is equal to zero (graphically, the slope of the function is =0)

Derivative: $g'(x) = dg(x)/dx$

If the function of interest is the production function, its derivative with respect to ***k*** representes the marginal productivity of capital (PMK)!

$c(k) = f(k) - \delta k$ is maximized when $c'(k) = dc(k)/dk = 0$

$\Rightarrow c'(k) = df(k)/dk - d(\delta k)/dk = f'(k) - \delta = 0$

Golden rule under Cobb-Douglas

Example: Cobb-Douglas production function: $\mathbf{Y = K^{\alpha} L^{(1-\alpha)}}$

$\Rightarrow$ $\mathbf{y = k^{\alpha}}$ (see previous slides)

PMK is $f'(\mathbf{k}) = \mathbf{d}f(\mathbf{k})/\mathbf{dk} = \alpha\, \mathbf{k}^{\alpha-1}$

$\Rightarrow$ According to the golden rule:

$\mathbf{c'(k)} = f'(\mathbf{k}) - \delta = \alpha\, \mathbf{k}^{\alpha-1} - \delta = 0$

$\Rightarrow$ the level of *k* of golden rule is: $\mathbf{k^{*}_{gold} = (\delta/\alpha)^{1/(\alpha-1)}}$

$\Rightarrow$ If $\mathbf{k^{*} = k^{*}_{gold}}$, the social welfare in the economy is maximized!

Numerical example

Consider a closed economy with:

- a Cobb-Douglas production funtion **$Y = K^{1/2} L^{1/2}$** $\Rightarrow$ **$y = f(k) = k^{1/2}$**
- a saving rate **s=0.6**
- a capital depreciation rate **δ = 0.2**

Steady state condition: **$k^*/f(k^*) = s/\delta$**

$\Rightarrow$ **$k^*/(k^*)^{1/2}$** $= 0.6/0.2 = 3 \Rightarrow$ **k^*** $= 3^2 = 9$

Golden rule: PMK = δ

Compute PMK: **$PMK = f'(k) = ½\, k^{(1/2)-1} = ½\, k^{-1/2}$**

Replace those data into the golden rule condition:

PMK = δ $\Rightarrow$ $½\, k^{-1/2} = 0.2 \Rightarrow$ **k^*_{gold}** $= (0.2 \times 2)^{-2} = (1/0.4)^2 = 6.25$

Remember: if **$k^* \neq k^*_{gold}$**, then social welfare is not maximized $\Rightarrow$ **s** has to be modified! (In this case it has to be reduced!)

Exercise 3

Draw the graph of the Solow's model assuming, for the sake of simplicity, no population growth (i.e. n=o).

- Show, by means of graphs, the transition towards the steady state of golden rule, when the economy is initially in a stady-state featuring a capital stock p.c. too low, if compared to one compatible with the golden rule.

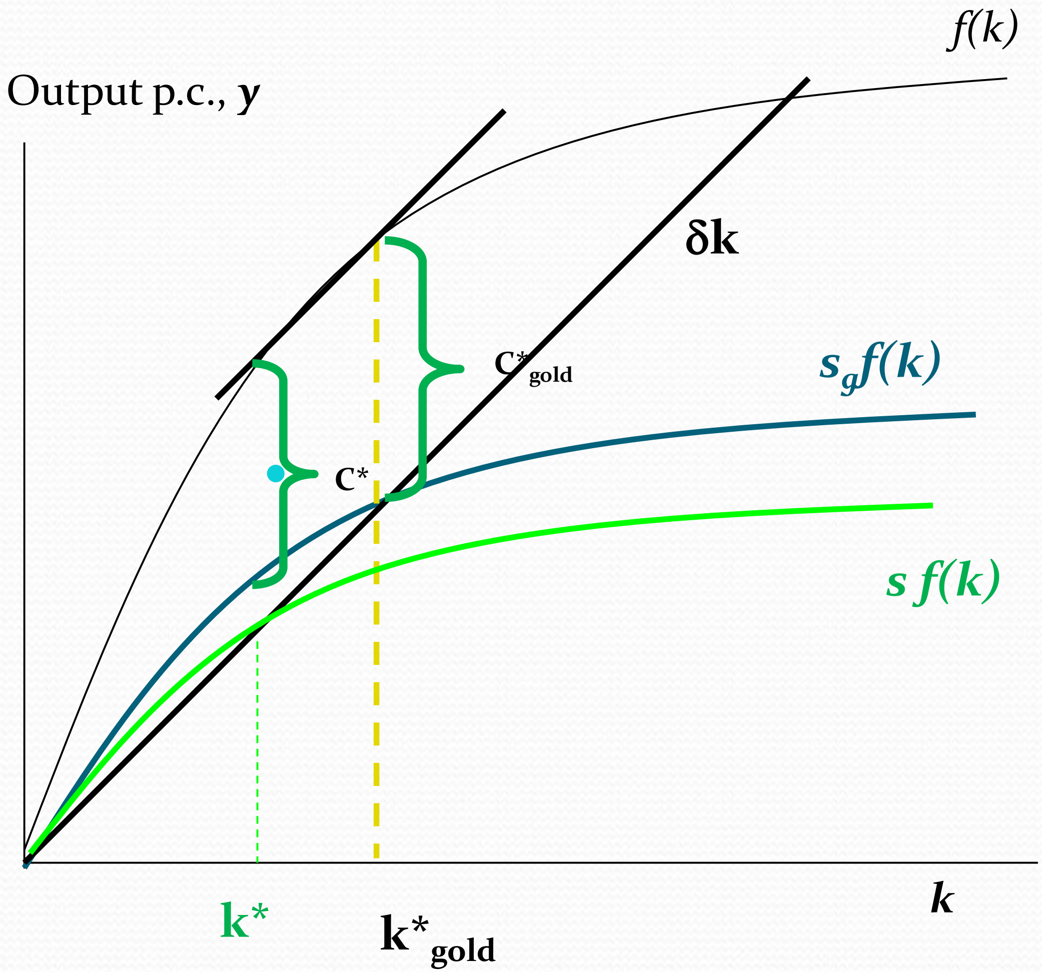
Ex.3

$k^* < k^*_{gold}$: if the capital stock p.c. is k^*, at that level PMK turns out to be larger than δ

s must be increased: *c* falls, *i* rises and this leads to an increase in *k* (gradually reducing PMK, until the level corresponding to δ)

The accumulation of capital leads to higher and higher levels of income and consumption

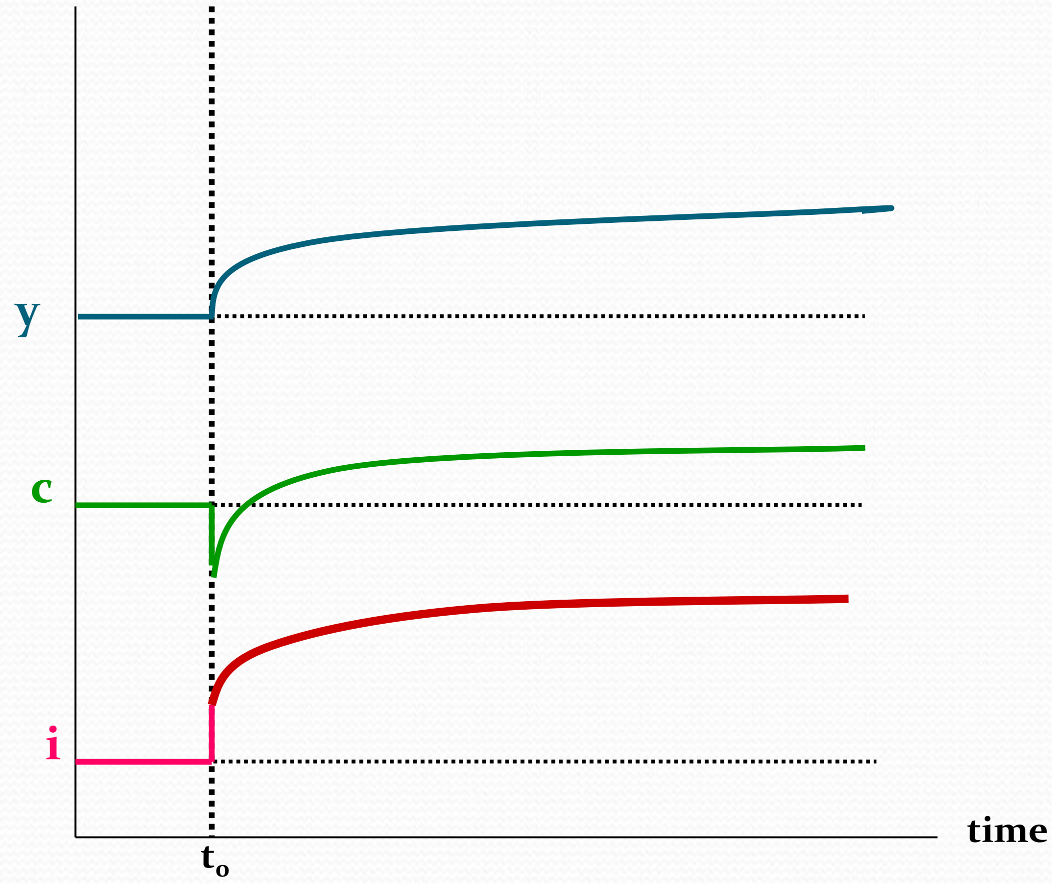
In the steady state of golden rule, the new equilibrium levels of consumption is higher than its initial level



Es.3 (dynamics)

At time t_o, s increases because the stock of capital p.c. is too low: $k^* < k^*_{gold}$

Consumption initially falls for investment to rise.



Ch. 9: Economic Growth II

Key- concepts

- Labor input measured in terms of efficiency units
- The role of technical progress in the Solow model

Efficiency units

The production function in the Solow model, denoted by

$F(K, L)$, can be generalized in such a way to account for variation in the efficiency in production:

$$F(K, L \times E)$$

E = efficiency of labor ⇒ $L \times E$ is the number of "effective workers" (technical progress is "labor augmenting", i.e. is equivalent to an increase in the labor force!)

The efficiency of labor, namely E, grows at rate g:

$$g = \frac{\Delta E}{E}$$

Efficiency of labor

All the relevant variables in the model can be expressed in terms of efficiency units:

Income: $y = Y/LE = f(Y/LE, 1)$

Capital: $k = K/LE$

Saving, Investment: $s\,y = s\,f(k)$

The change in the capital stock per efficiency unit of labor is: $(\delta + n + g)k$

$\delta\,k$ = depreciation

$n\,k$ = population growth

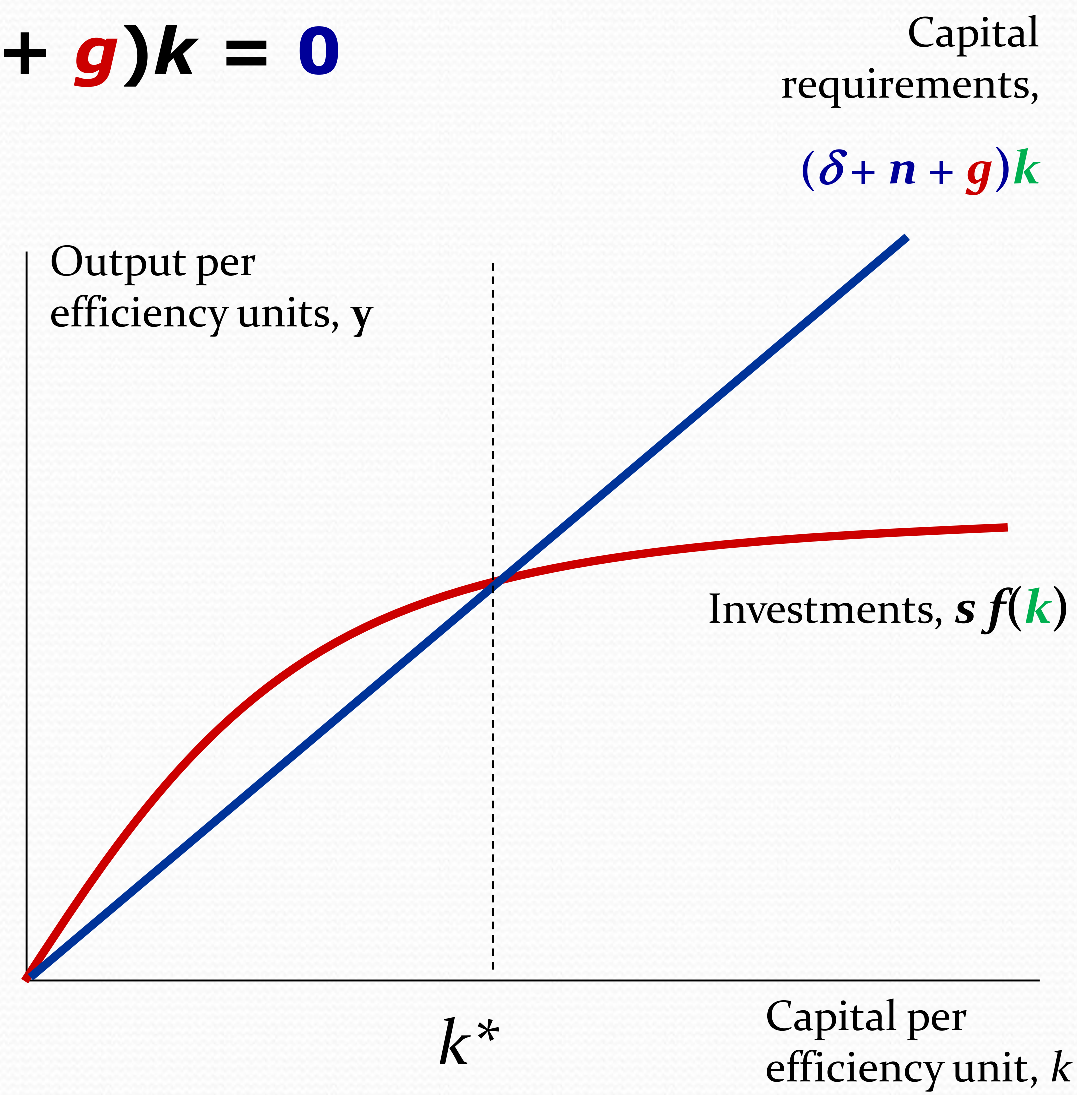
$g\,k$ = technical progress (workers get more efficient)

Technical progress

$\Delta k = s\,f(k) - (\delta + n + g)k = 0$

As in the baseline version of the Solow's model, the steady state is achieved when the stock of capital per efficiency unit does not vary any longer over time.

Note: in this case, what stops growing is not capital per worker, but capital per efficiency unit of labor!



Effects of technical progress

What are the growth rates of the different variables, once the steady state is achieved?

Variable	*Symbol*	*Growth rate in steady state*
Capital per efficiency unit	$k = K / (E \times L)$	0
Product per efficiency unit	$y = Y / (E \times L) = f(k)$	0
Product per worker (p.c.)	$Y/L = y \times E$	g
Total product	$Y = y \times (E \times L)$	$n + g$

Only technical progress can give rise to a persistant increase in the standards of living of an economy!

Golden rule with technical progress

The level of consumption in steady state is given by:

$$\boldsymbol{c}^* = \boldsymbol{y}^* - \boldsymbol{i}^*$$
$$= \boldsymbol{f}(\boldsymbol{k}^*) - (\delta + \boldsymbol{n} + \boldsymbol{g})\boldsymbol{k}^*$$

c^* is maximized when: $\boldsymbol{PMK} = \delta + \boldsymbol{n} + \boldsymbol{g}$

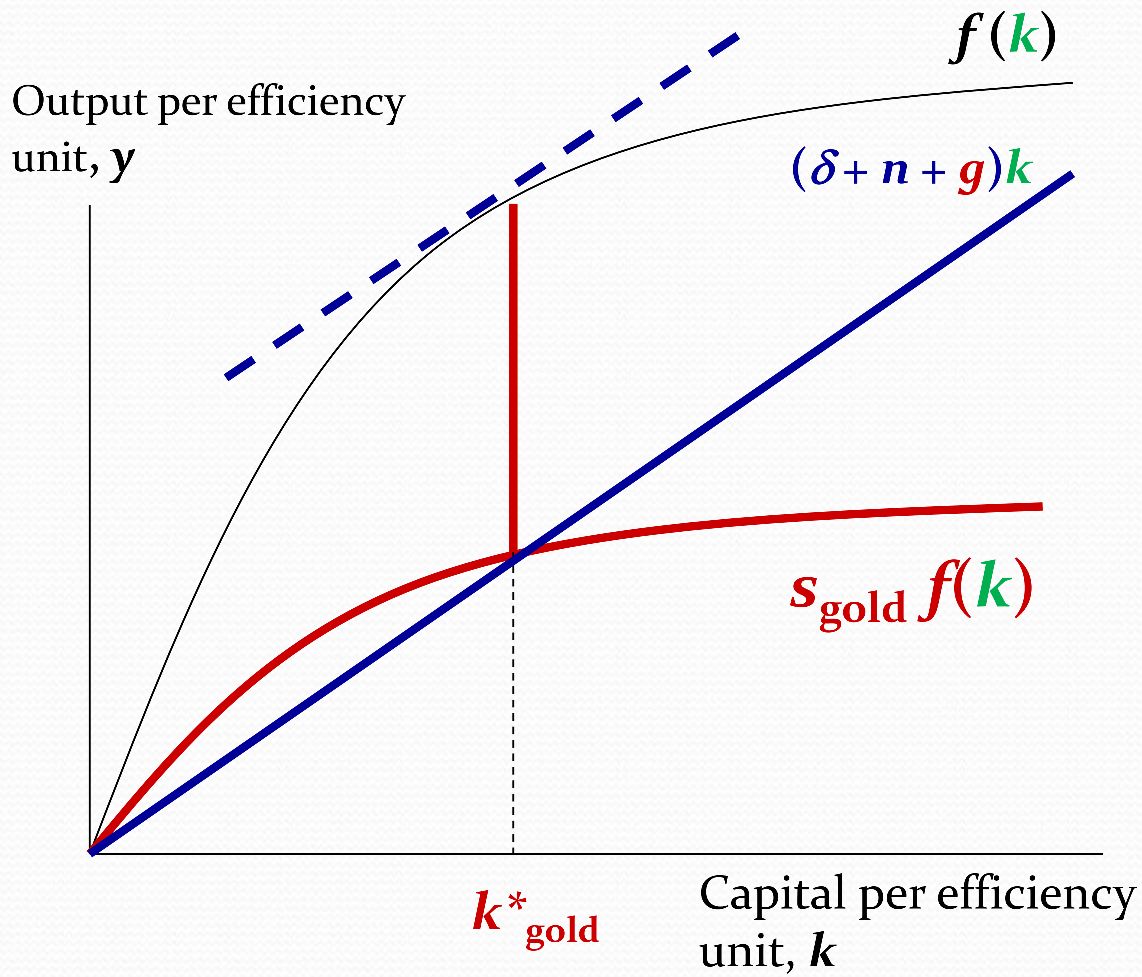
which means $\boldsymbol{PMK} - \delta = \boldsymbol{n} + \boldsymbol{g}$

where $\boldsymbol{PMK} = f'(k^*)$

Golden rule

In the graph, the stock of k associated with the steady state equilibrium of the golden rule is identified by the point at which the slope of the production function is equal to the slope of the line representing capital replacement requirements:

$$\boldsymbol{PMK = \delta + n + g}$$



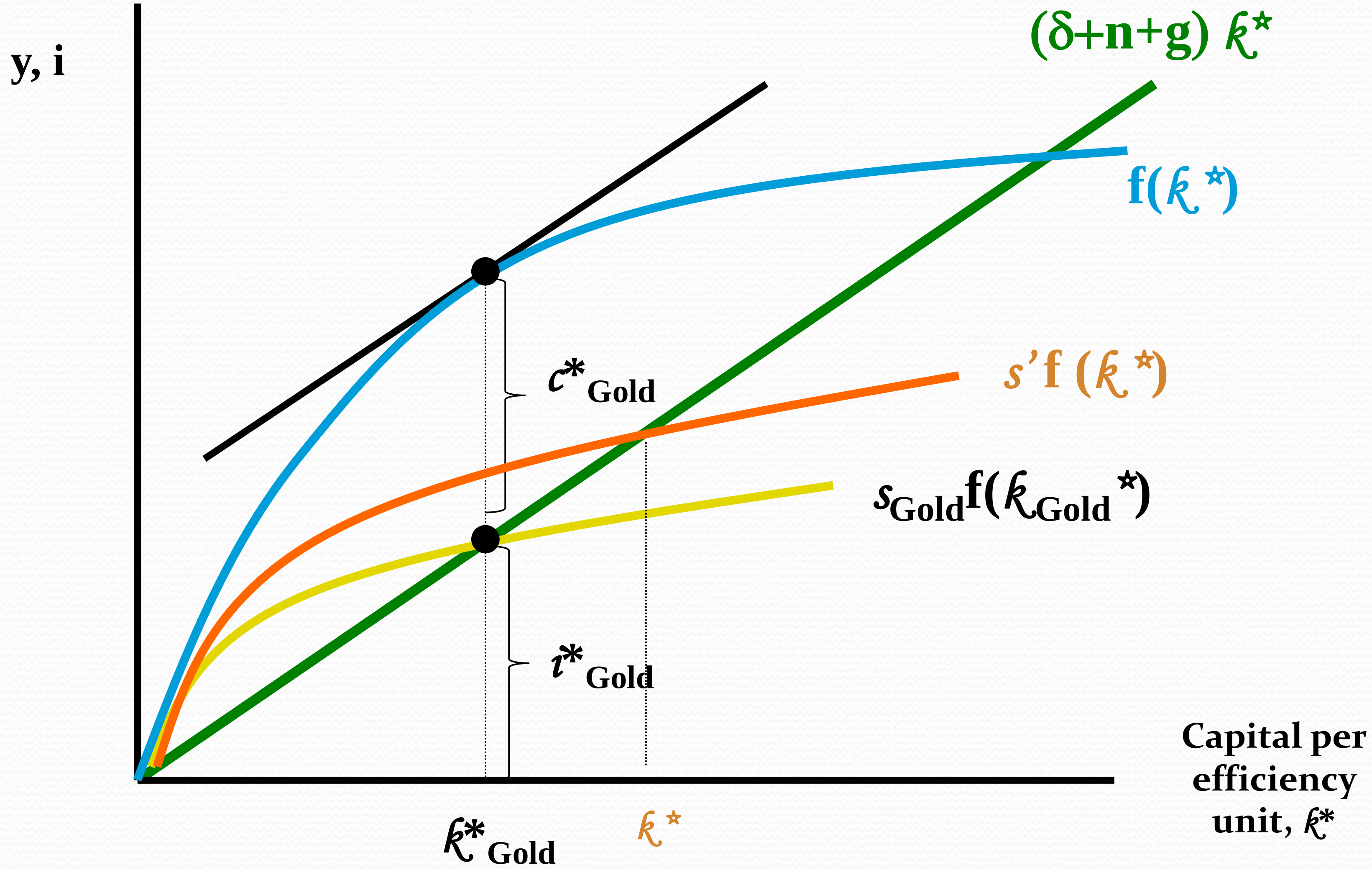
Exercise 4

Suppose that $k^* > k^*_{\text{gold}}$

«A reduction in the share of income devoted to saving (and therefore investment), i.e. a reduction in the saving rate *s*, will contribute to ensure higher standards of living»

Comment this statement and explain whether this is true or not, by using a Solow model with population growth and technical progress (of the type *labour augmenting*)

Ex. 4



y, i
(δ+n+g) k*
f(k*)
c*Gold
s'f (k*)
sGoldf(kGold*)
i*Gold
Capital per efficiency unit, k*
k*Gold
k*

Ex 4 - Dynamics

Suppose that k is initally too high, compared to the level compatible with the steady state of golden rule: ***s*** **must be reduced**

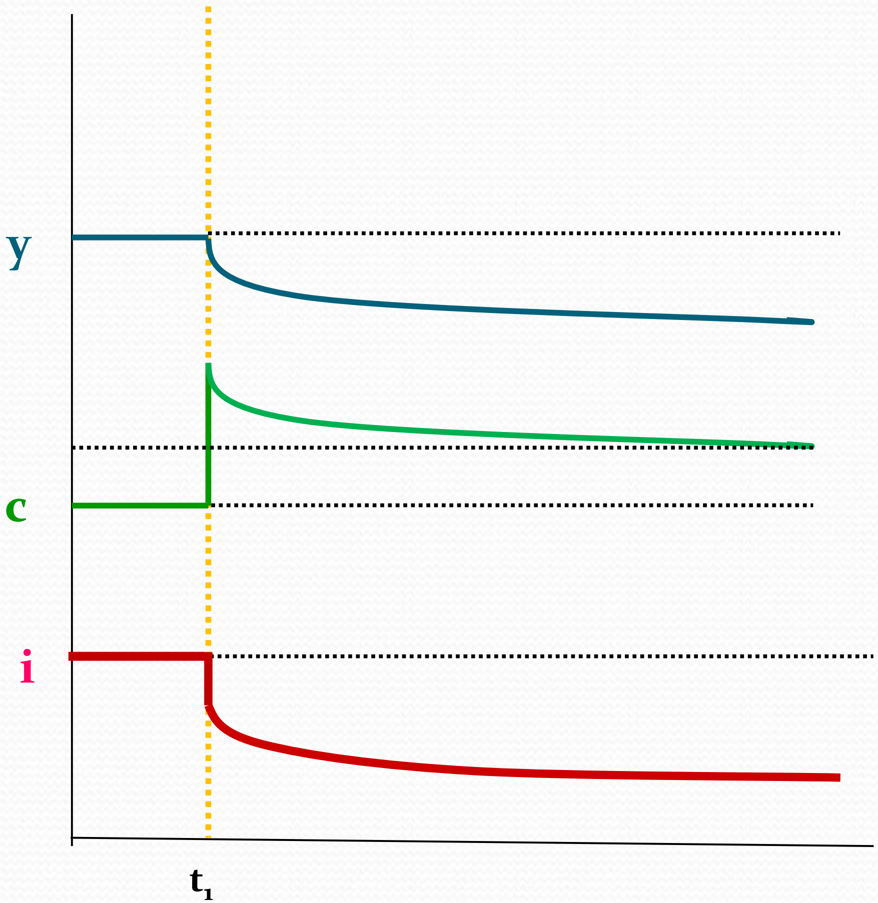
At t_1, the saving rate falls: investment falls and consumption rises

- Previously, at time t_0, $\boldsymbol{i=\delta k}$
- Now, at time t_1, $\boldsymbol{i < \delta k}$

This implies a reduction in k

➔ y, c and i fall until they reach their steady state levels of the golden-rule

Consumption suddenly jumps upwards, then starts falling, even if, at the end, it will stay at a higher level than its initial level



Exercise 5

Suppose that $k^* < k^*_{gold}$ and the saving rate must therefore rise in order to maximize social welfare

Illustrate the dynamic adjustment of the most relevant variables in the Solow model.

Explain who gains and who loses from moving away from the old equilibrium, towards the new one.

Ex.5 - Dynamics

Suppose that k is initially too low if compared to the level compatible with the steady state of golden rule: **s must increase**

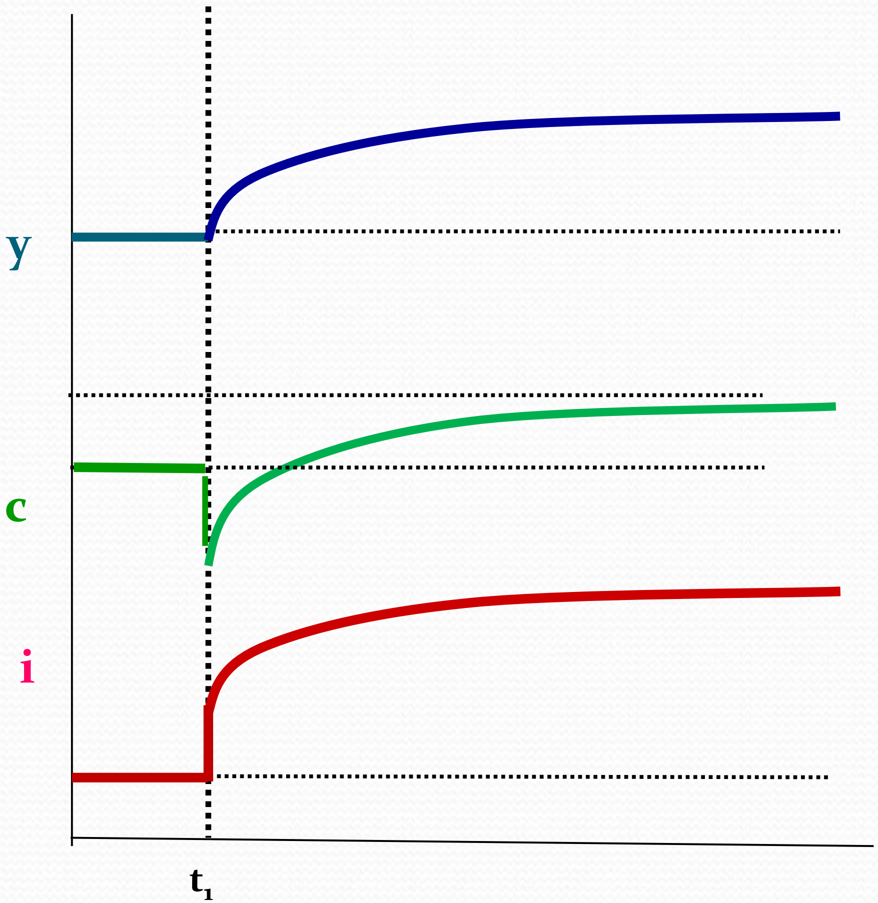
At t_1, the saving rate rises: investment increases and consumption falls

- Previously, at time t_0, $\boldsymbol{i=\delta k}$
- Now, at time t_1, $\boldsymbol{i > \delta k}$

This implies an increase in *k*

➔ *y*, *c* and *i* rise up to their steady state levels of golden-rule

Consumption level suddenly jumps downwards, then starts rising and, at the end, it will be higher than its initial level



Ex.5

Who loses and who gains?

- Suddenly, social welfare (if measured in terms of consumption) declines…
- … however, higher saving and investment will lead to a faster growth of the capital stock; the growth rate of Y and of y (=Y/L) will increase → the economy grows faster (note: we are in the short-run)
- In the new steady state, Y grows at rate $n+g$ while y grows at rate g (both growth rates do not depend on s).
- In the steady state of golden rule, the consumption level is higher, such that social welfare is increased for future generations

Absolute Convergence

- In the long run, GDP per worker (or per capita) converges to the same growth path in all countries. This implies that all countries converge to the ***same level of income per worker.***
- Furthermore, this would mean that all countries converge to the same level of capital-labor ratio, output per capita and consumption (k^*, y^*, c^*) with an ***equal growth rate g***.

Conditional Convergence

- A country's income per worker (or per capita) converges to a country specific long-run growth path, which is given by the ***basic structural characteristics*** of the country.
- Thus, the lower the countries original level of GDP per worker, the higher is the expected subsequent growth.
- This implies that the countries that start below their long run growth path are likely to ***grow at a faster pace***.
- Conditional convergence is a necessary but not a sufficient condition for absolute convergence.

Multiple choice

1. In the Solow model, the prediction of ***absolute convergence*** among countries means that…

a) poor countries will grow faster than rich ones.

b) poor countries will reach the same level of income per capita as rich countries in the long run.

c) poor countries will grow at the same rate as rich countries.

d) both answers a and b are correct.

Multiple choice

In the Solow model, the prediction of ***conditional convergence*** among countries means that…

a) The growth rate depends on the basic structural characteristics of the country .

b) poor countries will reach the same level of income per capita than rich countries.

c) poor countries will grow at the same rate as rich countries.

d) none of the above answers is correct.